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Preference based welfare analysis with unobserved heterogeneity

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Motivation

Preferences have two functions:

- explaining behaviour: positive analysis

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Preferences have two functions:

- explaining behaviour: positive analysis
- determining preference based welfare: normative analysis

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Motivation

Preferences have two functions:

- explaining behaviour: positive analysis
- determining preference based welfare: normative analysis

But ‘plug and play’ is more complicated than it seems...

Motivation

discrete choice over $X = \{x_1, \ldots, x_n\}$

$$U(x_k) = V(x_k) + \varepsilon_k$$

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Motivation

discrete choice over $X = \{x_1, \ldots, x_n\}$

$$U(x_k) = V(x_k) + \varepsilon_k$$

Should ε be incorporated in a welfare measure?

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Motivation

Welfare *differences*: include ε in calculation CV

- Dagsvik - Karlström (2005)

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Welfare *levels*:

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Welfare *levels*:

- Decoster - Haan (2014): only deterministic part

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- Carpantier - Sapata (2016): conditional means of the random terms

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Our contribution: include ε to determine distribution of welfare levels.

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Outline

Motivation

Theoretical results
- Stochastic equivalent income in an arbitrary bundle
- Stochastic equivalent income in a chosen bundle

Empirical illustration
- Data
- How to compare RVs?
- Stochastic EI vs. deterministic EI

Conclusion

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Labour context

Equivalent income in the labour context:

$$X = \{(w_0 = 0, h_0 = 0), (w_1, h_1), \ldots, (w_n, h_n)\}$$

$$U(C_k, h_k) = V(C_k, h_k) + \varepsilon_k$$

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Welfare measure: equivalent income

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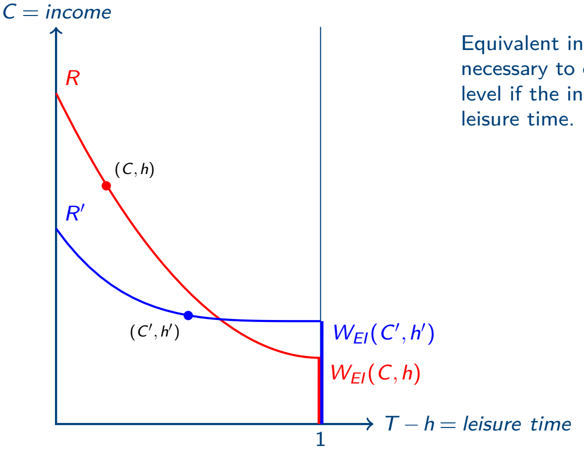
Equivalent income

Equivalent income:



Equivalent income measures income necessary to obtain the observed utility level if the individual would have full leisure time.

Equivalent income: deterministic and stochastic

Equivalent income in the labour context.

$$X = \{(w_0 = 0, h_0 = 0), (w_1, h_1), \ldots, (w_n, h_n)\}$$

$$U(C_k, h_k) = V(C_k, h_k) + \varepsilon_k$$

Option 1: "Deterministic"

$$V\left(W_{EI}(C_k, h_k), 0\right) \\ = V(C_k, h_k)$$

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Equivalent income: deterministic and stochastic

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Option 1: "Deterministic"

$$V\left(W_{EI}(C_k, h_k), 0\right) = V(C_k, h_k)$$

Option 2: "Stochastic"

$$V\left(W_{EI}(C_k, h_k), 0\right) + \varepsilon_0 = V(C_k, h_k) + \varepsilon_k$$

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Stochastic equivalent income

Consequences of Option 2: "Stochastic"

$$V\left(W_{EI}(C_k, h_k), 0\right) + \varepsilon_0 = V(C_k, h_k) + \varepsilon_k$$

- $W_{EI}(C_k, h_k)$ is a random variable

Stochastic equivalent income

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- distribution will depend on whether or not we condition on the actual choice

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In this paper, we

- determine the (un)conditional distribution of the equivalent income random variable

Stochastic equivalent income

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- $W_{EI}(C_k, h_k)$ is a random variable
- distribution will depend on whether or not we condition on the actual choice

In this paper, we

- determine the (un)conditional distribution of the equivalent income random variable
- empirically illustrate that stochastic EI $\neq$ deterministic EI

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Stochastic equivalent income

Setting:
Discrete choice over

$$X = \{(0,0),(w_1,h_1),\ldots,(w_n,h_n)\}$$

with

$$U_i(C_k,h_k) = V_i(C_k,h_k) + \varepsilon_k^i.$$

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Stochastic equivalent income

Setting:
Discrete choice over

$$X = \{(0,0),(w_1,h_1),\dots,(w_n,h_n)\}$$

with

$$U_i(C_k,h_k) = V_i(C_k,h_k) + \varepsilon_k^i.$$

where the ε_k^i are iid standard Gumbel distributed (EVI) i.e.

$$G(x) = \exp(-\exp(-x))$$

Stochastic equivalent income

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where the ε_k^i are iid standard Gumbel distributed ($\textsc{EVI}$) i.e.

$$G(x) = \exp(-\exp(-x))$$

Q: What is the (un)conditional distribution of $W_{EI}(C_k,h_k)$?

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Distribution of stochastic EI in an arbitrary bundle

Theorem (Unconditional distribution)

The stochastic equivalent income W_{EI}^{i} evaluated in an arbitrary bundle (C, h) equals the income when not working C_0 for $h = 0$.

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Distribution of stochastic EI in an arbitrary bundle

Theorem (Unconditional distribution)

The stochastic equivalent income W^i_{EI} evaluated in an arbitrary bundle (C, h) equals the income when not working C_0 for $h = 0$. For $h = h_k \neq 0$, it is a random variable distributed as follows:

$$P\left(W^i_{EI}(C_k, h_k) \leq y\right) = \frac{\exp\left(V_i\left(y, 0\right)\right)}{\exp\left(V_i\left(y, 0\right)\right) + \exp\left(V_i(C_k, h_k)\right)}.$$

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Distribution of stochastic EI in an arbitrary bundle

$$P\Big(W_{EI}^{i}(C_k, h_k) \leq y\Big) = \frac{\exp\left(V_i\left(y, 0\right)\right)}{\exp\left(V_i\left(y, 0\right)\right) + \exp\left(V_i(C_k, h_k)\right)}.$$

Interpretation:

probability that an individual would choose option $(y, 0)$ over (C_k, h_k) when the choice set consists of those two bundles.

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Outline

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Distribution of stochastic EI in a chosen bundle

Theorem (Conditional distribution)

The stochastic equivalent income W_{EI}^i evaluated in a chosen bundle (C, h) equals the income when not working C_0 for $h = 0$.

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Distribution of stochastic EI in a chosen bundle

Theorem (Conditional distribution)

The stochastic equivalent income W^i_{EI} evaluated in a chosen bundle (C,h) equals the income when not working C_0 for $h=0$. For $h=h_k\neq 0$, it is a random variable distributed as follows:

$$P\Big(W^i_{EI}(C_k,h_k)\leq y \mid k \text{ is chosen}\Big)=$$

$$\begin{cases} 0 & \text{if } y<C_0,\\ \frac{\exp(V_i(y,0))-\exp(V_i(C_0,0))}{\exp(V_i(y,0))+\left(\sum_{h_j\in H\setminus\{0\}}\exp\big(V_i(C_j,h_j)\big)\right)} & \text{otherwise.}\end{cases}$$

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Distribution of stochastic EI in a chosen bundle

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Note (1):

$$P\left(W_{EI}^{i}(C_k,h_k) \leq y \mid k \text{ is chosen }\right) =$$

$$P\left(W_{EI}^{i}(C_j,h_j) \leq y \mid j \text{ is chosen }\right).$$

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Distribution of stochastic EI in a chosen bundle

$$P\left(W^i_{EI}(C_k,h_k) \leq y \mid k \text{ is chosen}\right) = \begin{cases} 0 & \text{if } y < C_0, \\ \frac{\exp(V_i(y,0)) - \exp(V_i(C_0,0))}{\exp(V_i(y,0)) + \left(\sum_{h_j \in H\setminus\{0\}} \exp\left(V_i(C_j,h_j)\right)\right)} & \text{otherwise.} \end{cases}$$

Note (1):

$$P\left(W^i_{EI}(C_k,h_k) \leq y \mid k \text{ is chosen}\right) = P\left(W^i_{EI}(C_j,h_j) \leq y \mid j \text{ is chosen}\right).$$

$\Rightarrow$ only information on (the valuation of) the choice set is needed

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Distribution of stochastic EI in a chosen bundle

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Note (2):

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If $y < C_0: \ V_i(C_0, 0) + \varepsilon^i_0 > V_i(y, 0) + \varepsilon^i_0$

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If $y < C_0 : \; V_i(C_0, 0) + \varepsilon_0^i > V_i(y, 0) + \varepsilon_0^i = V_i(C_k, h_k) + \varepsilon_k^i$

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Outline

Empirical illustration

Empirical illustration has two goals:

- how to compare individuals with each other if their welfare is a random variable?

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Empirical illustration

Empirical illustration has two goals:

- how to compare individuals with each other if their welfare is a random variable?
- how does the stochastic EI compare with the deterministic EI?

Empirical results

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Outline

Data

Data for the empirical illustration:

- SILC 2015
- singles
- between 18 and 64, available for the labour market
- Self-employed individuals and employers excluded
- no extra adults available for the labour market are allowed

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Model estimated on it:

- Preferences estimated as in Capéau et al. (2018)
- gender specific Box–Cox utility function
- marginal rates of substitution which depend on age, education, region and the number of children

Data

Table: Descriptive statistics SILC subsample

Description	Female	Male
Age (years)	43.0	41.5
Dependent children (%)		
0 – 3 years	5.0	0.7
4 – 6 years	6.2	1.4
7 – 9 years	7.8	0.9
Experience (years)	16.6	18.8
Education (%)		
Low	22.1	19.6
Middle	36.1	39.0
High	41.8	41.4
Residence (%)		
Brussels	20.2	17.3
Flanders	45.4	48.3
Wallonia	34.4	34.4
Participation rate (%)	67.9	73.7
Hours worked (hours/week)		
Unconditional	24.0	29.8
Conditional on working	35.3	40.4
Hourly wage (euro)	20.4	21.2
Disposable income (euro/month)	2123.1	2345.9
Number of observations	644	526

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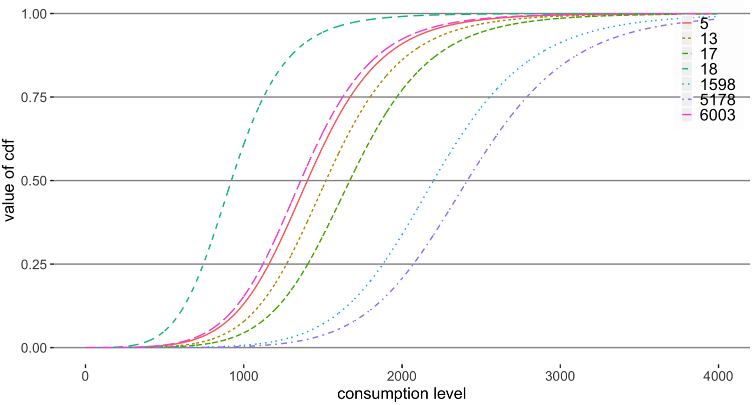
Outline

Empirical illustration: how to compare RVs?

Figure: Cdf of stochastic equivalent income for some working individuals



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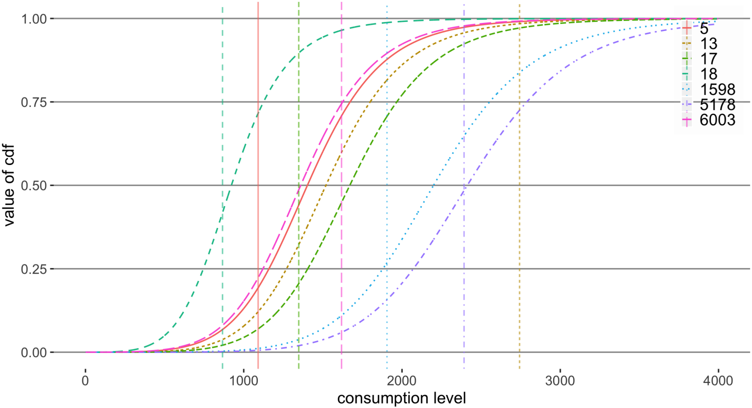
Empirical illustration: how to compare RVs?

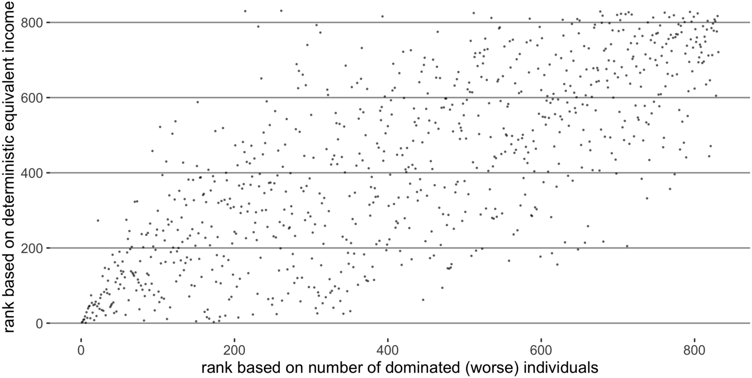
Figure: Cdf of stochastic equivalent income for some working individuals compared to deterministic equivalent income (dashed lines)



Outline

Stochastic EI vs. deterministic EI

Figure: Rank plot



dominating analysis RURO

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Outline

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Conclusion

In this paper, we

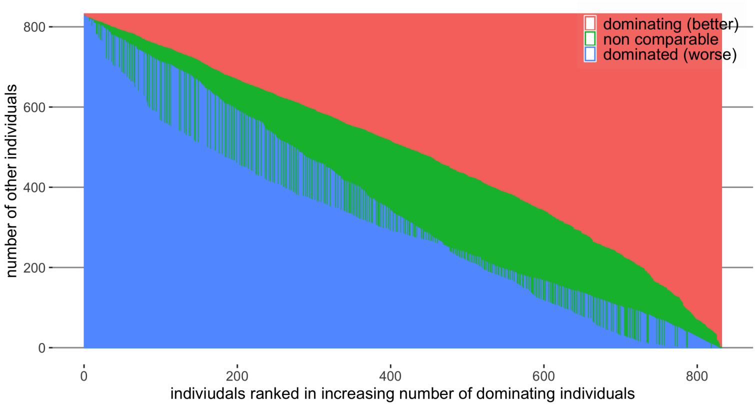
- developed the concept of Stochastic EI
- derived the (un)conditional distribution of stochastic EI in discrete choice context
- empirically illustrated the relevance of the concept by proving it differs considerably from determinisitc EI

Preference based welfare analysis with unobserved heterogeneity

Thank you!

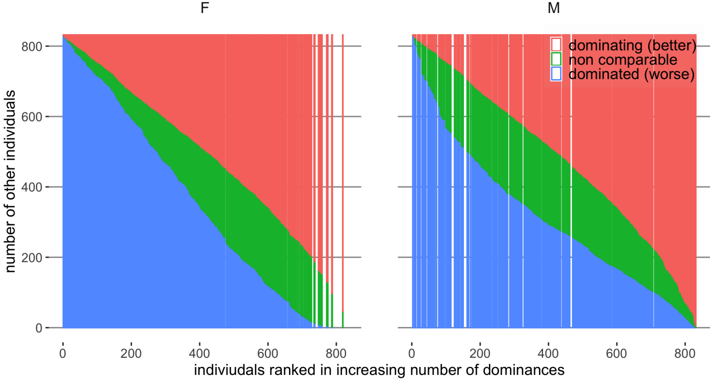
Empirical illustration: how to compare RVs?

Figure: The number of stochastic dominances per individual



Empirical illustration: how to compare RVs?

Figure: The number of stochastic dominances per individual split by gender



F versus all | M versus all | F versus F | M versus M

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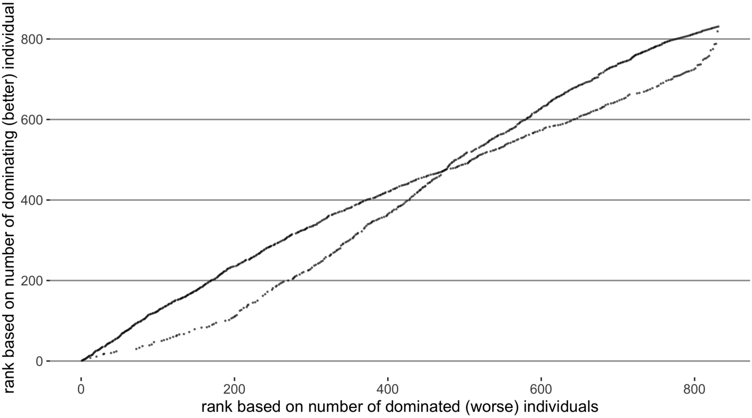
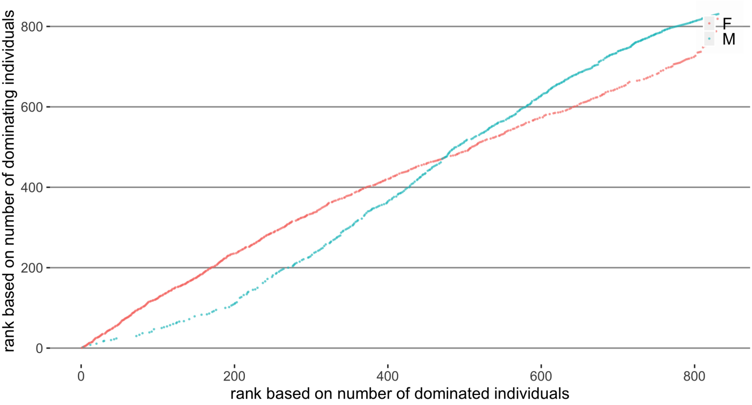
Empirical illustration: how to compare RVs?

Figure: Rank plot



Empirical illustration: how to compare RVs?

Figure: Rank plot by gender



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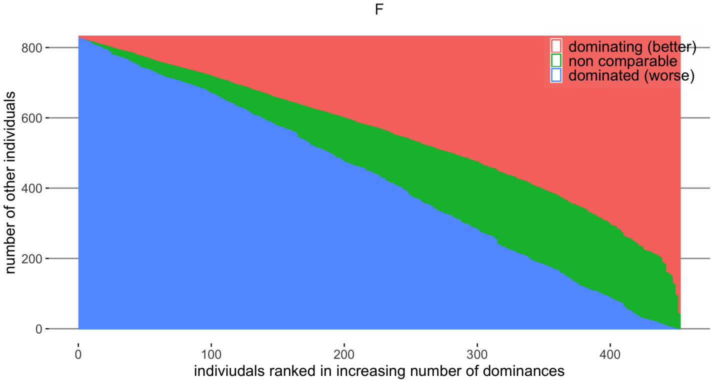
Stochastic EI vs. deterministic EI

Figure: The number of stochastic dominances per individual - females vs all



all versus all
M versus all
F versus F
M versus M

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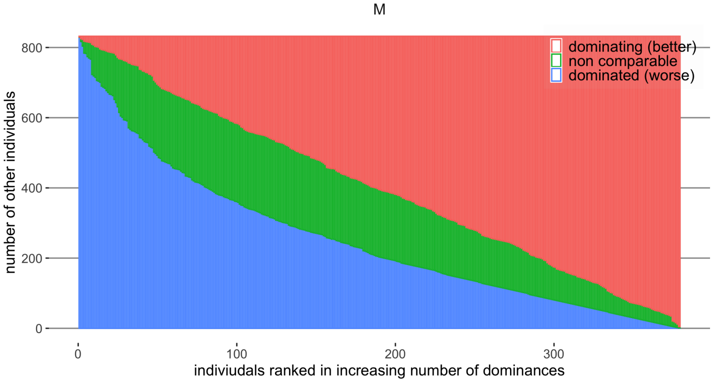
Empirical illustration: how to compare RVs?

Figure: The number of stochastic dominances per individual - males vs all



all versus all | F versus all | F versus F | M versus M

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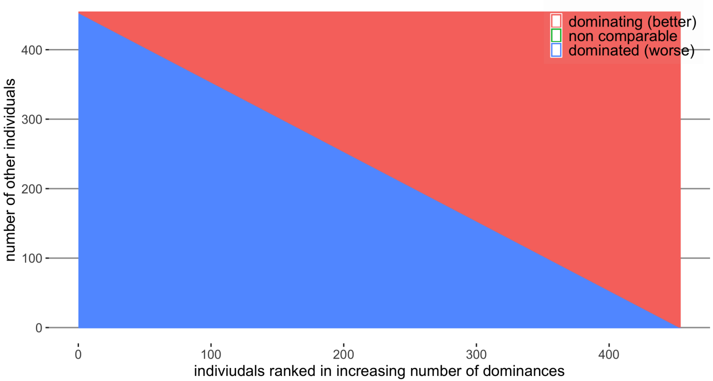
Empirical illustration: how to compare RVs?

Figure: The number of stochastic dominances per individual - females vs females



all versus all | F versus all | M versus all | M versus M

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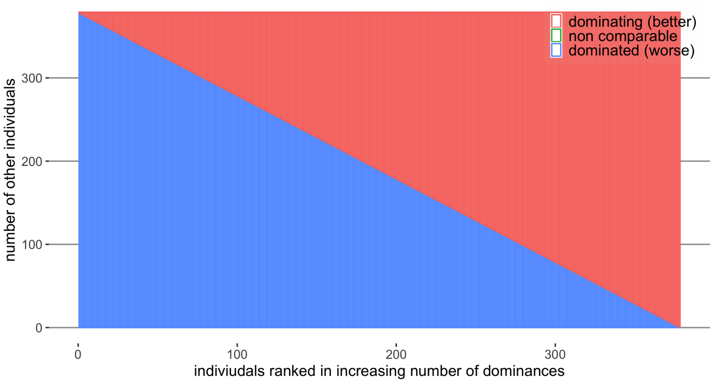
Empirical illustration: how to compare RVs?

Figure: The number of stochastic dominances per individual - males vs males



all versus all | F versus all | M versus all | F versus F

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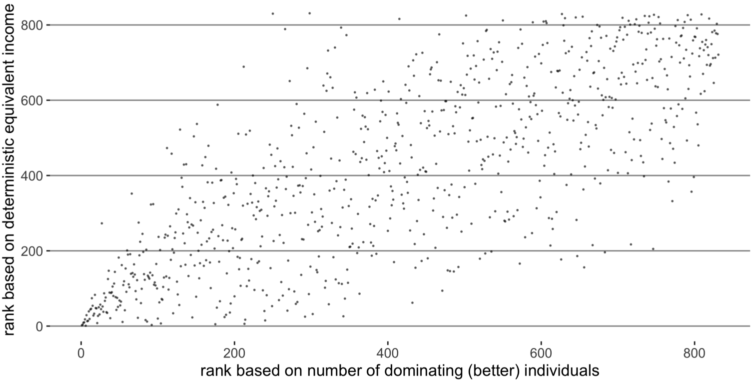
Stochastic EI vs. deterministic EI

Figure: Rank plot



dominated analysis

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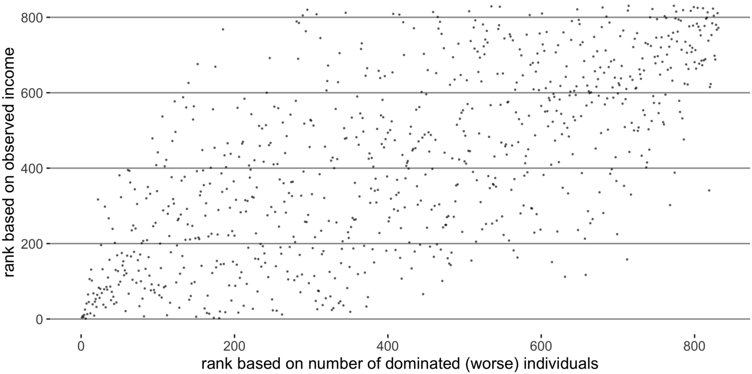
Stochastic EI vs. deterministic EI

Figure: Rank plot stochastic versus observed consumption



dominated

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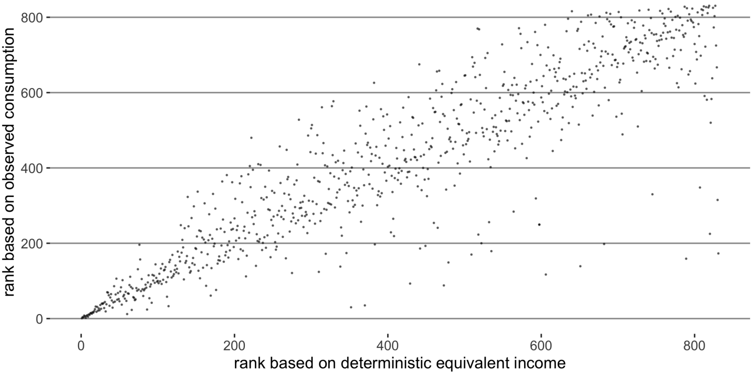
Stochastic EI vs. deterministic EI

Figure: Rank plot deterministic versus observed consumption



dominated

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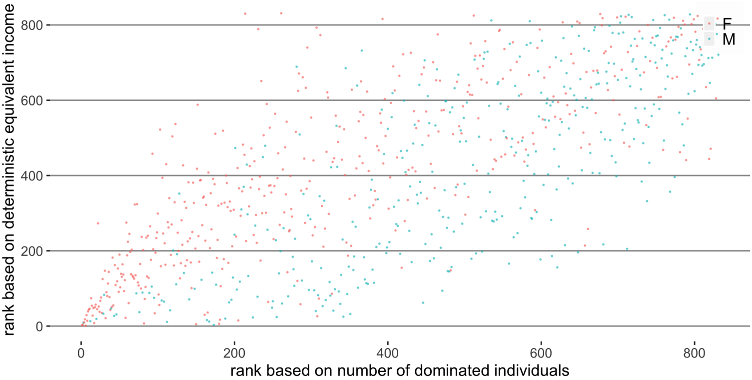
Stochastic EI vs. deterministic EI

Figure: Rank plot by gender



dominated

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Stochastic EI vs. deterministic EI

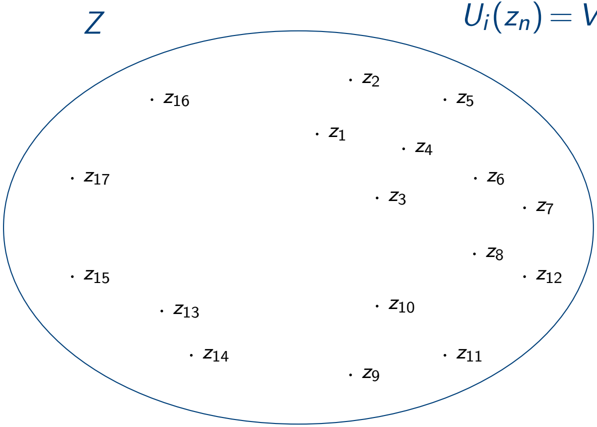
Table: Rank differences stochastic vs. deterministic

	stochastic		deterministic	
rank quintiles	F	M	F	M
$(1, 167]$	128	38	88	78
$(167, 333]$	116	50	97	69
$(333, 499]$	104	62	98	68
$(499, 665]$	74	92	89	77
$(665, 831]$	29	137	79	87

dominated

RURO: the idea

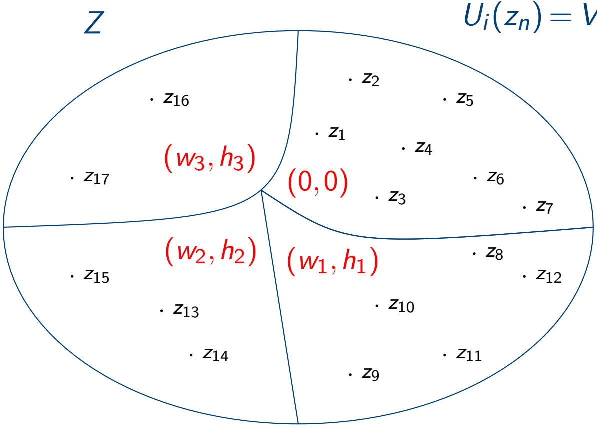
$$U_i(z_n) = V_i\Big(C(z_n), h(z_n)\Big) + \varepsilon(z_n)$$



dominated

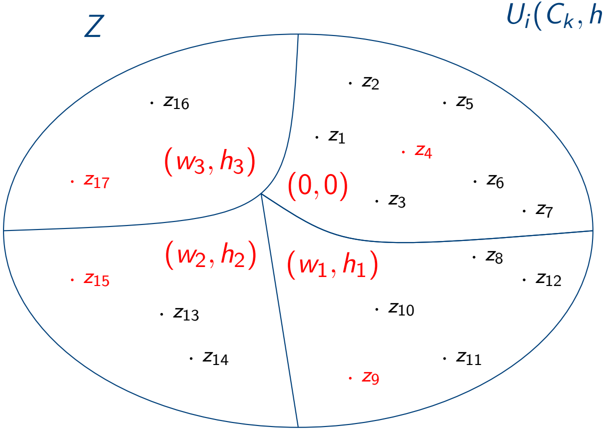
RURO: the idea

$$U_i(z_n) = V_i\Big(C(z_n), h(z_n)\Big) + \varepsilon(z_n)$$



RURO: the idea

$$U_i(C_k, h_k) = \max_{n \in Z_k} U_i(z_n)$$



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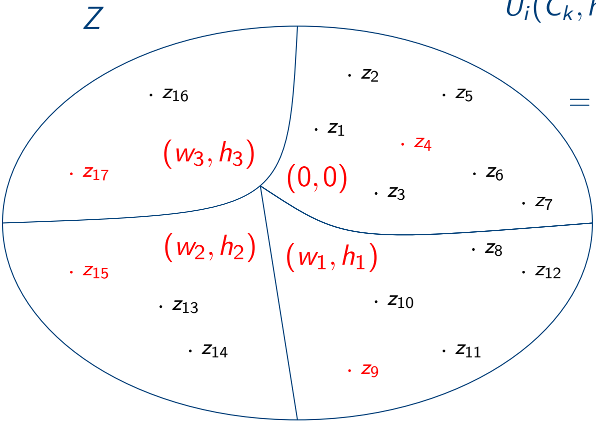
RURO: the idea



$$U_i(C_k, h_k) = \max_{n \in Z_k} U_i(z_n)$$

$$= \max_{n \in Z_k} \{ V_i\Big(C(z_n), h(z_n)\Big) + \varepsilon(z_n) \}$$

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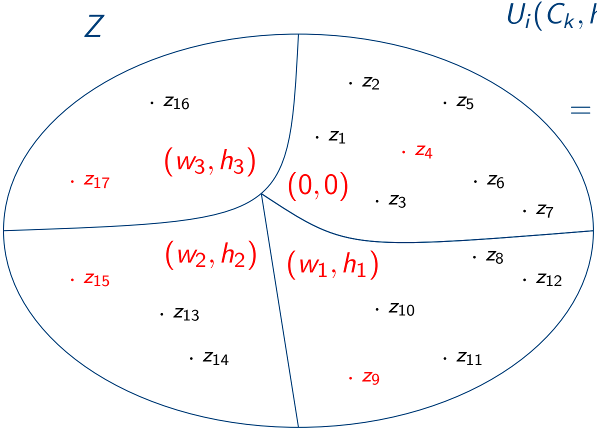

$$U_i(C_k, h_k) = \max_{n \in Z_k} U_i(z_n)$$

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$$= V_i(C_k, h_k) + \varepsilon_k$$

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- individuals choose jobs, not only labour hours

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RURO, summary:

- individuals choose jobs, not only labour hours
- discrete choice, but
- idiosyncratic choice sets, which are draws from individual specific random variables
- argument for Gumbel distributed random terms

Empirical illustration

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Distribution of EI in arbitrary bundle (RURO)

Theorem (Unconditional distribution - RURO)

The stochastic equivalent income W_{EI}^{i} evaluated in an arbitrary set of bundles $\mathscr{B}$ equals the income when not working C_0 for $\mathscr{B} = \{0\}$.

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Distribution of EI in arbitrary bundle (RURO)

Theorem (Unconditional distribution - RURO)

The stochastic equivalent income W^i_{EI} evaluated in an arbitrary set of bundles $\mathscr{B}$ equals the income when not working C_0 for $\mathscr{B} = \{0\}$. For $\mathscr{B} \neq \{0\}$, it is a random variable distributed as follows:

$$P\left(W^i_{EI}\left(\mathscr{B}\right) \leq y\right) = \frac{\exp\left(V_i\left(y, 0\right)\right)}{\exp\left(V_i\left(y, 0\right)\right) + \exp(\tilde{\mu}^i_W - \tilde{\mu}^i_0) \exp\left(V_i(\mathscr{B})\right)}.$$

where $\exp\left(V_i(\mathscr{B})\right) = \int_{\mathscr{B}} g_i(w, h) \exp\left(V_i(C(w, h), h)\right) dw\, dh$

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Distribution of EI in chosen bundle (RURO)

Theorem (Conditional distribution - RURO)

The stochastic equivalent income W^i_{EI} evaluated in a chosen bundle in a set $\mathscr{B}$ equals the income when not working C_0 for $\mathscr{B} = \{0\}$.

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Distribution of EI in chosen bundle (RURO)

Theorem (Conditional distribution - RURO)

The stochastic equivalent income W^i_{EI} evaluated in a chosen bundle in a set $\mathscr{B}$ equals the income when not working C_0 for $\mathscr{B} = \{0\}$. For $\mathscr{B} \neq \{0\}$, it is a random variable distributed as follows:

$$P\left(W^i_{EI}\left(\mathscr{B}\right) \leq y \mid \text{ a bundle in } \mathscr{B} \text{ is chosen}\right) =$$

$$\begin{cases} 0 & \text{if } y < C_0, \\ \frac{\exp(V_i(y,0)) - \exp(V_i(C_0,0))}{\exp(V_i(y,0)) + \exp(\tilde{\mu}^i_W - \tilde{\mu}^i_0)\exp(V_i(X))} & \text{otherwise.} \end{cases}$$

where $\exp\left(V_i(X)\right) = \int_X g_i(w,h) \exp\left(V_i(C(w,h),h)\right) dw\, dh$

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