

Pension systems in an overlapping generations framework: The impact of introducing a pension sustainability factor on inequality and growth

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Economics

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- **Research interest:**

What is the impact of modifying the pension system on income distribution and growth when individuals are heterogeneous?

Motivation (short-lived subsidize long-lived)

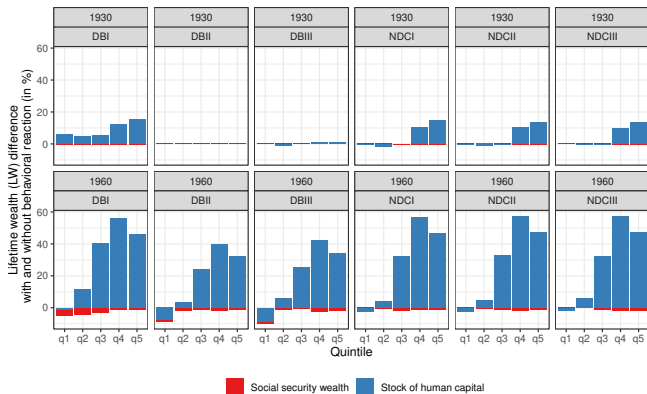


Figure 1: Impact of behavior on lifetime wealth by income quintile and pension system. US males, mortality regimes of birth cohorts 1930 (top panels) and 1960 (bottom panels). Source: Sánchez-Romero, Lee, Prskawetz (2019)

Notes: DB-I=DB Flat replacement

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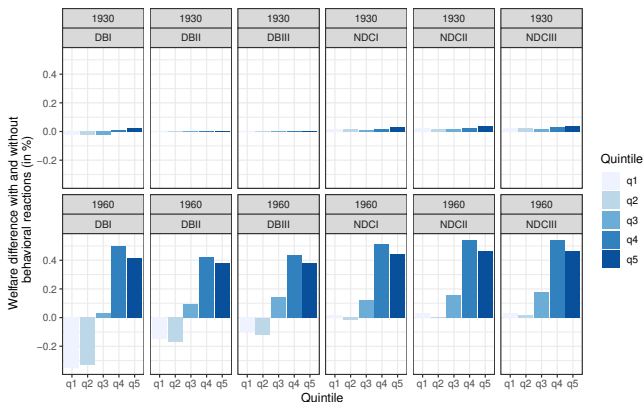


Figure 2: Impact of behavior on welfare by income quintile and pension system. US males, mortality regimes of birth cohorts 1930 (top panels) and 1960 (bottom panels). Source: Sánchez-Romero, Lee, Prskawetz (2019)

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- Literature:

- Correlation between education, health, labor market, and length of life: Chakraborty (2004), Chakraborty and Das (2005)
- Pension system without cohort heterogeneity: Keuschnigg and Keuschnigg (2004), Fisher and Keuschnigg (2010), Jaag et al. (2010), Fehr et al. (2013), etc...
- Redistributive properties of the pension system with differences in life expectancy: ... Fehr, Kallweit, and Kindermann (2012, 2017), NAS (2015), Pestieau and Ponthiere (2016), Sanchez-Romero and Prskawetz (2017), Haan et al. (2019), Laun et al. (2019), Sanchez-Romero et al. (2019), Holzmann et al. (2020), Lee and Sanchez-Romero (2020)

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- **Model:** Large scale computable general equilibrium model with two productive sectors (health and final good) and a social security system calibrated to the Austrian economy
 - **Heterogeneity:**
 - **Exogenous:** Ability, health, and effort of attending school (parental background)
 - **Endogenous:** Educational attainment and life expectancy

Parametric components of past and present Austrian pension systems

- **Contribution period**

- Benefits are calculated according to an ordered vector of the highest past labor incomes

Let $\mathbf{p} \in \mathbb{R}^{py}$, where $p_1 > p_2 > p_3 > \dots > p_{py}$

- Pensionable income years (py)
- Accrual rate $\phi^P(z)$
- Pension base Increment (PBI): \rightarrow Pension base (PB)

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• Benefit period

- Early retirement (R_e), normal retirement (R_n), and late retirement (R_l)
- Years contributed (yc) vs. Targeted years worked (yw)
- Penalties and rewards for early and late retirement (∂)
- Pension replacement rate ($f_{rep}(z)$)

Austrian pension system: Contribution period

- Pension points (pp) dynamics

$$pp' = \hat{R}(e, l)pp + PBI(z, l)$$

Capitalization index $\hat{R}(e, l) = (1 - \alpha_R(l)) \cdot \frac{1+\tilde{r}}{\pi(e)} + \alpha_R(l) \cdot 1$

Fraction retired $\alpha_R(l)$

Minimum pension benefits $pp_0 > 0$

Pension base increment $PBI(z, l) = \phi^P(z) \max \{y_l(l) - p_{py}, 0\}$

Accrual rate $\phi^P(z) = \frac{1.00}{py(z)}$

Pensionable income years

$$py(z) = \begin{cases} 15 & \text{for } z < 1955 \\ 15 + (z - 1955) & \text{for } 1955 \leq z \leq 1985 \\ 45 & \text{for } z > 1985 \end{cases}$$

- Pension benefit (b):

$$b_{za}(pp, l) = f_{rep}(z)pp \cdot sf(z + a)\partial(yc, yw, a)$$

Pension repl. rate $f_{rep}(z) = 0.8$ for $z > 1918$

Pension points pp

Sustainability factor $sf(z + a)$ (Benchmark =1)

Adjustment factors $\partial(\text{year contrib.}, \text{year worked}, \text{ret. age})$

Model: Household problem

Given a random set of endowments $\xi = (\theta_h, \phi_e, d_0) \in \Xi$, an educational level $e \in \mathbf{E}$, and the set of state variables $\mathbf{x} = \{a, h, d, pp\}$, our individual chooses consumption (c), labor (l), and health spending (m) that maximize the following Bellman equation:

$$J(\mathbf{x}; e, \xi) = \max_{c, l, m} \{F(d)U(c, l; e, \phi_e) + \beta\pi'(e)J'(\mathbf{x}'; e, \xi)\}$$

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subject to

$$a' = R(e, \tau^r)a + (1 - \tau^l) \left[(1 - \tau^s) \overbrace{w\epsilon(e)hl}^{y_l(l)} + b\alpha_R(l) \right] - (1 + \tau^c)c - p^m m,$$

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boundary conds $a_{z0} = a_{zT} = 0$, $h_{z0} = h_0$, $d_{z0} = d_0$

Solution: Control variables

- **First-order conditions:**

Consumption: $U_c(c, l; e, \phi_e) = \frac{\beta \pi(e) \lambda_a' (1 + \tau^c)}{F(d)}$

Health investment: $m = \left(\beta_d \theta_d \gamma_d \frac{-\varphi_D'}{p^m} \right)^{\frac{1}{1-\gamma_d}}$

Labor:

$$\frac{U_l(c, l; e, \phi_e)}{U_c(c, l; e, \phi_e)} = \underbrace{(1 - \tau^E) w \epsilon(e) h}_{\text{labor incentives (intensive)}} + \underbrace{\frac{(1 - \tau^l) \frac{\partial(b\alpha_R(l))}{\partial l} + \varphi_P' \frac{\partial \hat{R}(l)}{\partial l}}{1 + \tau^c}}_{\text{labor incentives (extensive)}}$$

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- **Education decision:** $e^+(\xi) = \arg \max_{e \in E} J_0(\mathbf{x}_0; e, \xi)$

- Input factors clearing

$$K_t = \sum_{z=t-\Omega}^t \int_{\Xi} N_{z,t-z}(\xi) \mathbf{a}_{z,t-z}^+(\xi) d\Phi(\xi),$$

$$L_t = \sum_{z=t-\Omega}^t \int_{\Xi} N_{z,t-z}(\xi) \epsilon_{t-z}(\mathbf{e}_z^+(\xi)) \mathbf{h}_{z,t-z}^+(\mathbf{d}_{z,t-z}^+(\xi)) \mathbf{l}_{z,t-z}^+(\xi) d\Phi(\xi)$$

Equilibrium conditions

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- Market goods clearing

Health goods: $M_t = \Gamma_H(1 - \ell_t)L_t,$

Final goods: $C_t + G_t + K_{t+1} = K_t^{\alpha_K} (\Gamma_t \ell_t L_t)^{1-\alpha_K} + (1 - \delta)K_t$

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- **Government**

Social security: $SS_t = \sum_{z=t-\Omega}^t \int_{\Xi} N_{z,t-z}(\xi) \mathbf{b}_{z,t-z}^+(\xi) \alpha_R (\mathbf{l}_{z,t-z}^+(\xi)) d\Phi(\xi)$
70% $SS_t = \tau_t^s w_t L_t$ and

Public budget: $G_t + 30\% SS_t = \tau_t^c C_t + \tau_t^l w_t L_t + \tau_t^r r_t K_t,$

Calibration (heterogeneous endowments)

Bayesian melding (Poole and Raftery, 2000)

1. Draw a sample of size 5 000 values of $\xi = (\theta_h, \phi_e, d_0)$ values from $\mathcal{U}([0.02, 0.30] \times [5, 35] \times [0.03, 0.06])$
2. For each ξ_i sampled, we run the model $M(\xi_i)$ to obtain v_i
3. We estimate $q_1^*(v)$ using a kernel density estimator
4. We construct the importance sampling weights
$$\hat{w}_i = \left(\frac{q_2(M(\xi_i))}{q_1^*(M(\xi_i))} \right)^{1-\alpha} L_1(\xi_i) L_2(M(\xi_i))$$
5. Sample 200 triplets from the discrete distribution $(\xi_i, \hat{w}_i) \Rightarrow \Phi(\xi)$

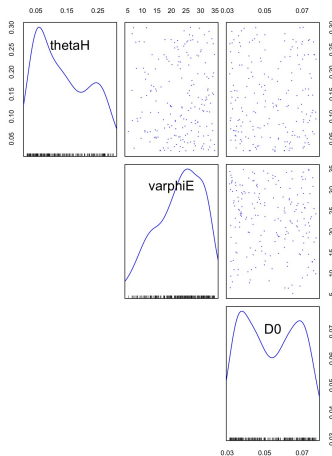


Figure 3: Posterior distributions: Endowments $\Phi(\xi)$

In-sample performance (preliminary)

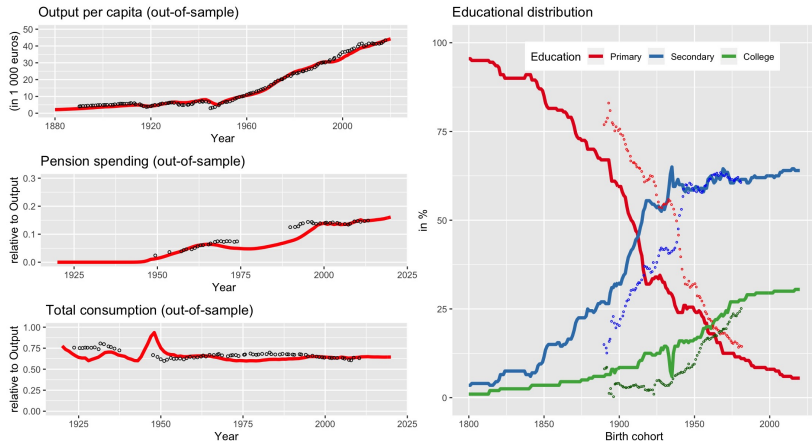


Figure 4: In-sample performance of the model: Benchmark.

Source: Data (dots) taken from Statistik Austria, WIC human capital database, and own calculations.

- **Benchmark** (*status quo*)

Social security: $SS_t = \sum_{z=t-\Omega}^t \int_{\Xi} N_{z,t-z}(\xi) \mathbf{b}_{z,t-z}^+(\xi) \alpha_R(\mathbf{I}_{z,t-z}^+(\xi)) d\Phi(\xi)$
 $70\% SS_t = \tau_t^s w_t L_t$ and

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- **Sustainability factor**

Social security: $SS_t = \sum_{z=t-\Omega}^t \int_{\Xi} N_{z,t-z}(\xi) \mathbf{b}_{z,t-z}^+(\xi) \alpha_R(\mathbf{I}_{z,t-z}^+(\xi)) d\Phi(\xi)$

$$\begin{cases} sf(t) = 1, & 70\%SS_t = \tau_t^s w_t L_t & \text{if } \tau_t^s < 22\%, \\ sf(t) < 1, & 70\%SS_t = 22\%w_t L_t & \text{otherwise,} \end{cases}$$

Public budget: $G_t + 30\%SS_t = \tau_t^c C_t + \tau_t^l w_t L_t + \tau_t^r r_t K_t$,

Pension spending and social contributions

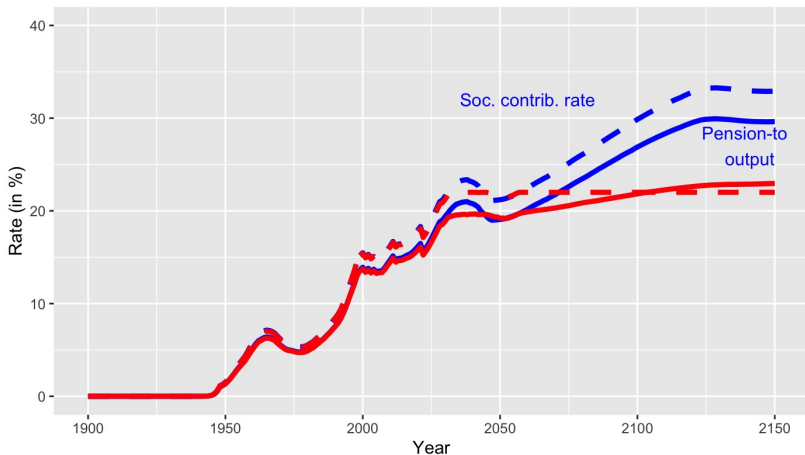


Figure 5: Pension spending to output ratio (solid) and social contribution rate (dashed) under the **Benchmark** and the **Sustainability factor**

Redistribution: Internal rate of return (IRR)

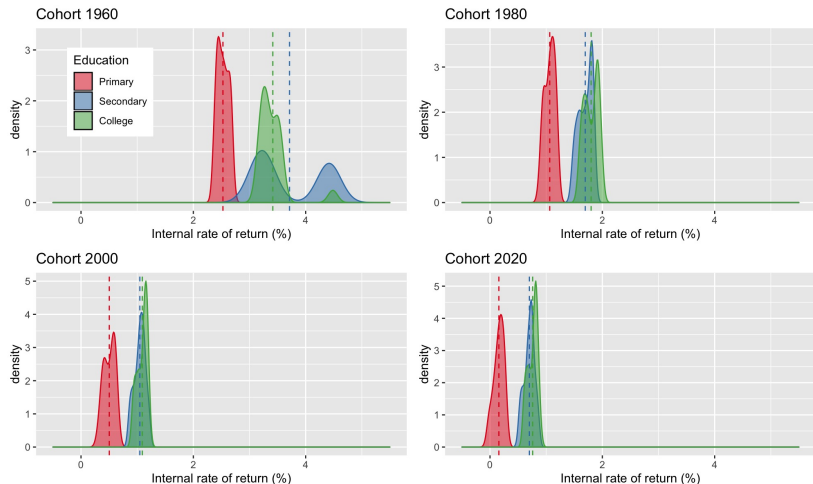


Figure 6: Internal rate of return of the Austrian pension system for cohorts born between 1960 and 2020: Case **Benchmark** No diff LE

Redistribution: Internal rate of return (IRR)

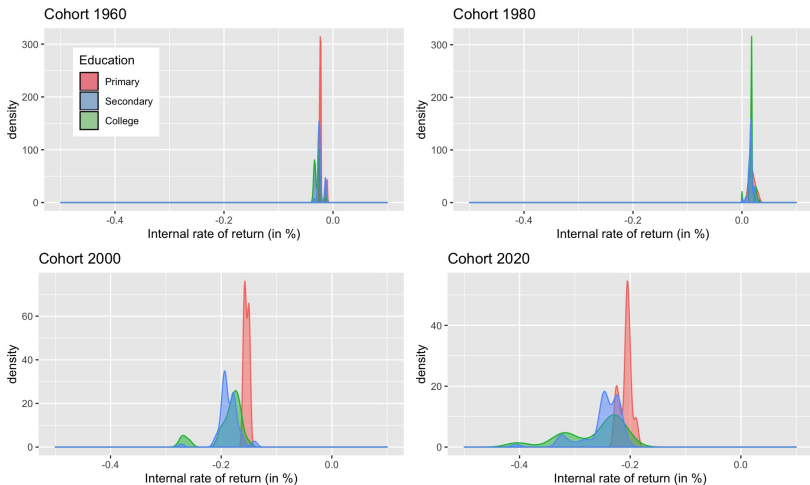


Figure 6: Internal rate of return of the Austrian pension system for cohorts born between 1960 and 2020: Case **Sustainability factor minus Benchmark**

No diff LE

Retirement age (Benchmark)

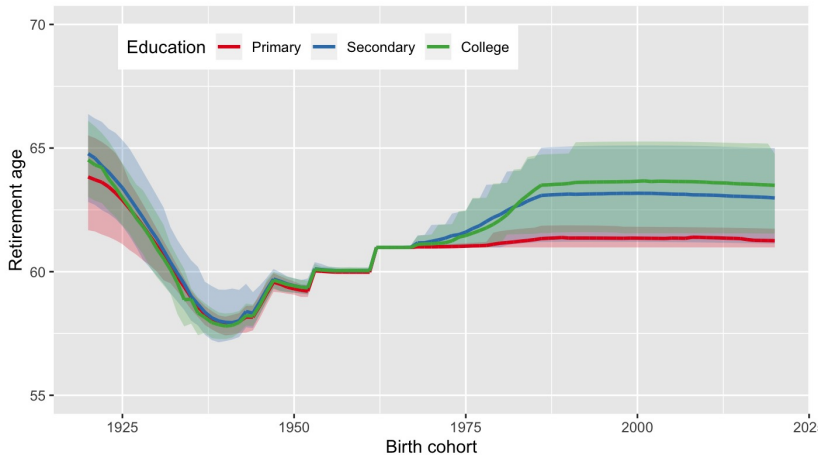


Figure 7: Retirement age. Case, Benchmark

Impact of the sustainability factor

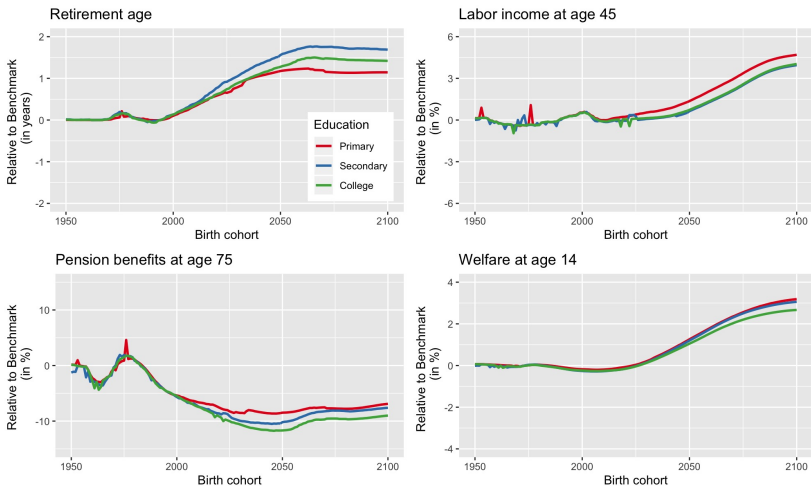


Figure 8: Impact of the sustainability effect. **Sustainability factor minus Benchmark**

Impact on labor: Effective labor income tax (τ^E)

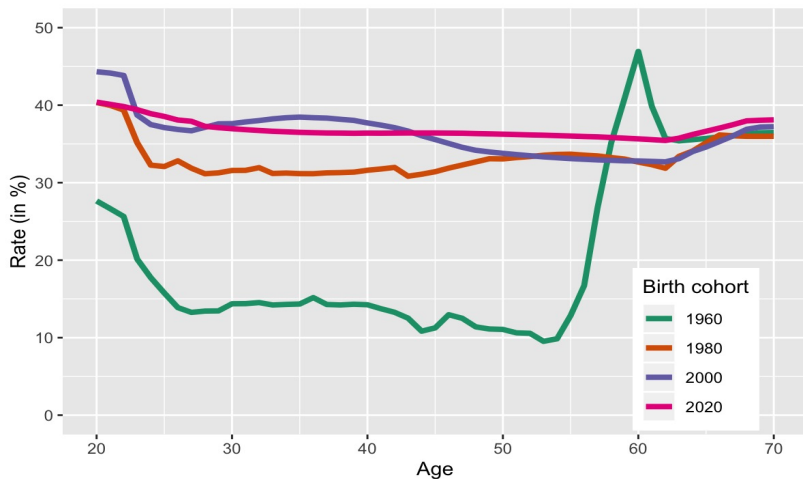


Figure 9: Effective labor income tax rate for Austrian cohorts born in 1960, 1980, 2000 and 2020: Case **Benchmark**

Impact on labor: Effective labor income tax (τ^E)

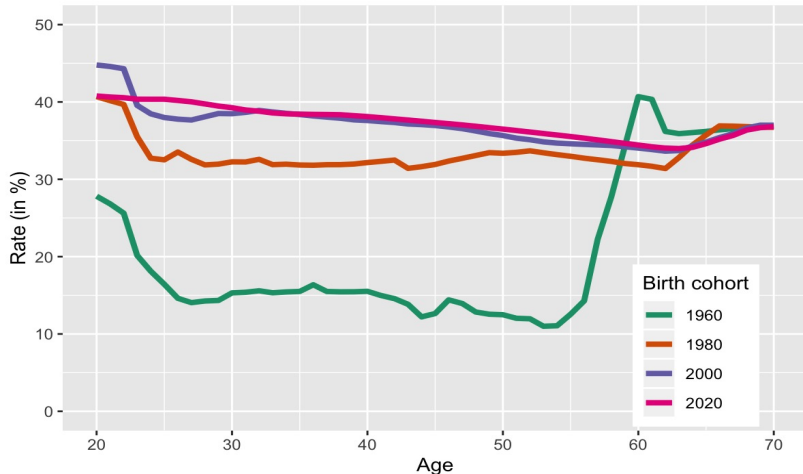


Figure 9: Effective labor income tax rate for Austrian cohorts born in 1960, 1980, 2000 and 2020: Case **Sustainability factor**

Growth: Impact of the reform on per capita income

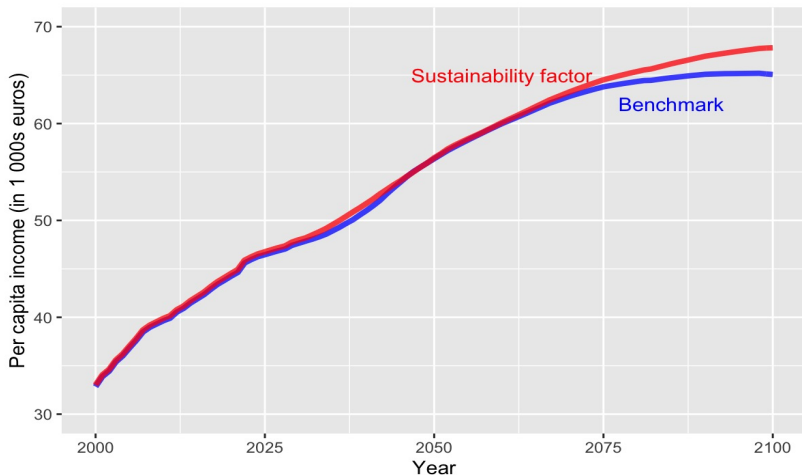


Figure 10: Output per capita (productivity detrended), Austria 2000–2100

- **Model:** We have constructed a CGE-OLG model with realistic demography that includes economic-demographic heterogeneity and is consistent at the micro and macro level.
- **Impact on the internal rate of return:**
 - Individuals with higher education enjoy a greater internal rate of return from the pension system
 - Lowering the pension replacement rate significantly reduces the internal rate of return of the pension system for all education groups, although more than proportional for the highest income groups

- **Impact on the effective labor income tax:**
 - Introducing a pension sustainability factor does not substantially modify the effective labor income tax
- **Impact on retirement:**
 - Reducing pension benefits increases the average retirement age
- **Next step:** Introducing a sustainability factor that takes into account life expectancy heterogeneity

Thank you!

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Life cycle profiles

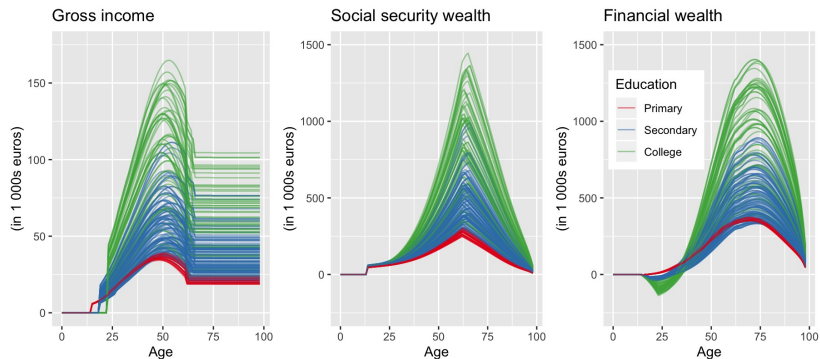


Figure 11: Life cycle profiles for the cohort born in 1980: Case **Benchmark**

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Internal rate of return (IRR): No Difference in Life Expectancy by Education

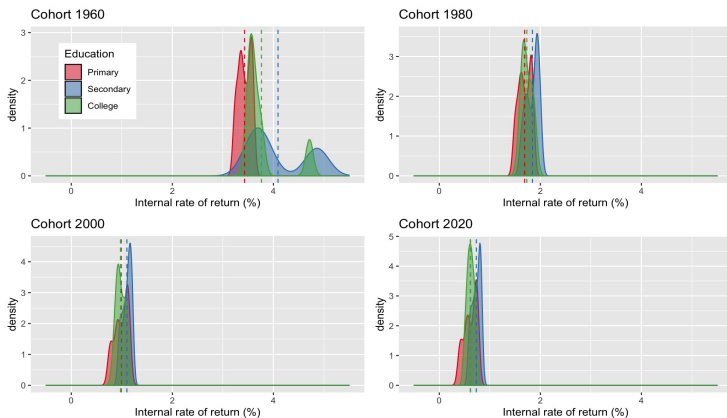


Figure 12: Internal rate of return of the Austrian pension system for cohorts born in 1960, 1980, 2000 and 2020: Case **Benchmark** Diff LE

Internal rate of return (IRR): No Difference in Life Expectancy by Education

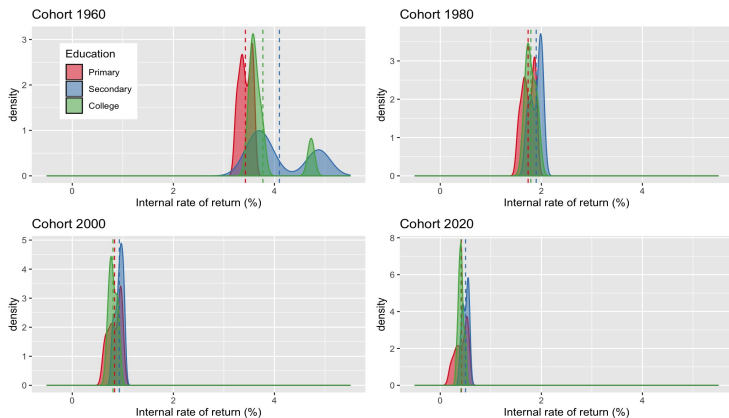


Figure 12: Internal rate of return of the Austrian pension system for cohorts born in 1960, 1980, 2000 and 2020: Case **Sustainability factor** Diff LE

Heterogeneous fertility and mortality

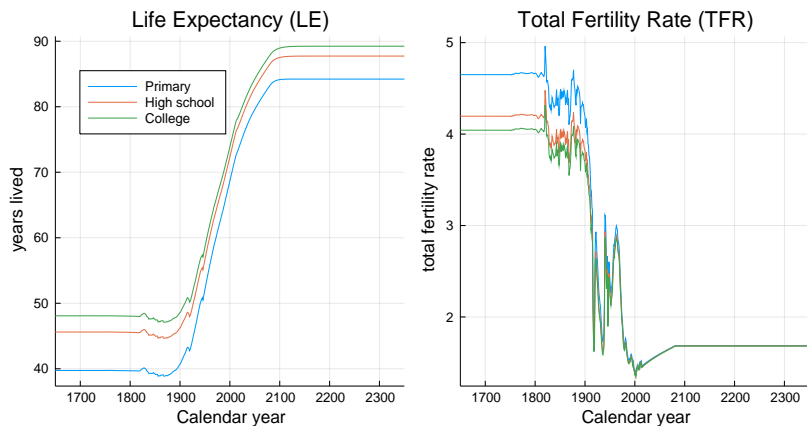


Figure 13: Life expectancy and total fertility rates, Austria 1650–2350

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Solution: State variables

- Value of a unit of human capital ($\varphi_H = \lambda_H/\lambda_A$)

$$\varphi_H' = \frac{R(e, \tau^r)}{R_h} \varphi_H - \frac{y_I(l)(1 - \tau^H)}{R_h}$$

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$$\varphi_H' = \frac{R(e, \tau^r)}{R_h} \varphi_H - \frac{y_I(l)(1 - \tau^H)}{R_h}$$

- Value of reducing health deficits ($\varphi_D = -\lambda_D/\lambda_A$)

$$\varphi_D' = \frac{R(e, \tau^r)\varphi_D}{(1 + \beta_d)} + \frac{\partial \delta(d)}{\partial d} \frac{\varphi_H' h}{1 + \beta_d} - \frac{\partial F(d)/\partial d}{F(d)} \frac{U(1 + \tau^c)}{U_c(1 + \beta_d)}$$

Solution: State variables

- **Value of a unit of human capital** ($\varphi_H = \lambda_H/\lambda_A$)

$$\varphi_H' = \frac{R(e, \tau^r)}{R_h} \varphi_H - \frac{y_I(l)(1 - \tau^H)}{R_h}$$

- **Value of reducing health deficits** ($\varphi_D = -\lambda_D/\lambda_A$)

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- **Value of pension points** ($\varphi_P = \lambda_P/\lambda_A$)

$$\varphi_P' = \frac{R(e, \tau^r)}{\hat{R}(l)} \varphi_P - (1 - \tau^l) \frac{\partial b\alpha_R(l)}{\partial \mathbf{pp}}$$

Table 1: Tax rates and contribution rates in selected years, Austria

Simulation	Year	Soc. Sec. contribution rate τ_t^s	Consumption tax rate τ_t^c	Labor income tax rate τ_t^l	Capital income tax rate τ_t^r
Benchmark	2020	0.176	0.204	0.227	0.114
	2060	0.224	0.203	0.241	0.114
	2100	0.297	0.204	0.262	0.114
Sustainability factor	2020	0.205	0.204	0.227	0.114
	2060	0.203	0.203	0.240	0.114
	2100	0.194	0.194	0.249	0.114

Short-hand notation

- Instantaneous utility

$$U(c, l; e, \xi) = \eta(e) \log \frac{c}{\eta(e)} - \phi_e \mathbf{1}_{\{a < e\}} - \alpha_L \frac{l^{1+\sigma_L^{-1}}}{1 + \sigma_L^{-1}} + \alpha_R(l) \nu_0 L E(e)^{\nu_1}$$

- Capital net interest rate

$$R(e, \tau^r) = (1 + r(1 - \tau^r))/\pi(e)$$

- Rate of return to E years of education

$$R_h = 1 + (\gamma_h/h) \mathbf{1}_{\{a < e\}} \theta_h(h)^{\gamma_h} - \delta(d)$$

- Effective labor income tax

$$\tau^E = (\tau^c + \tau^l(1 - \tau^s) + \tau^s - \phi^P(z) \varphi_{P'} \mathbf{1}_{\{y_l(l) > p_{py}\}})/(1 + \tau^c)$$

- Effective human capital tax

$$\tau^H = \tau^l(1 - \tau^s) + \tau^s - \phi^P(z) \varphi_{P'} \mathbf{1}_{\{y_l(l) > p_{py}\}}$$

Computational strategy

- Given the initial endowments of each cohort $\xi = \{\theta_h, \phi_e, d_0\}$, we draw for every cohort a sample of size $n = 200$ from $\mathcal{U}([0.02, 0.30] \times [5, 35] \times [0.03, 0.06])$

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- Given the population, the fertility rates and mortality rates by education $\{\pi_{z,a}(e), f_{z,a}(e)\}_{e \in \mathbf{E}, t=1650, \dots, 2350, a=0, \dots, 100}$ see LE and TFR

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Step 1 Start with an initial value for the co-state variables, prices, taxes, and contributions

Step 2 Calculate the household problem for all $\xi \in \Xi$ and cohorts

Step 3 Average all household profiles see profiles

Step 4 Multiply the average household profiles by the population

Step 5 Calculate the aggregate inputs and the total pension spending

Step 6 Adjust prices $\{r_t, w_t\}$ that close the capital and labor markets

Step 7 Calculate the new social contribution rates and tax rates that balanced the public budget

Step 8 Calculate $\text{Err} = \sqrt{\sum_{t=1650}^{2350} (r^{\text{supply}} - r^{\text{demand}})^2}$

Step 9 If $\text{Err} < 0.01$, then finish; otherwise go to Step 1

Table 2: Model parameters

IES on consumption	σ_C	1.000	Human capital	β_1	EU-SILC
IES on labor	σ_L	0.400		β_2	EU-SILC
weight on labor	α_L	86.17		h_0	1.0
Health disutility	ϵ	0.050		γ_h	0.65
	\bar{D}	0.031		δ_h	0.15
Retirement utility	v_0	-2.50			
	v_1	373.92	Health deficits	β_d	0.0430
				θ_d	0.0025
Production				α_d	0.0110
Health care sector	A_h	1		γ_d	0.0200
Final good sector	α_y	0.375			
	g_y	Bergeaud et al. (2016)			